2023 Spring PHYS2320 電磁學（Electromagnetism）　Final　(double sides)

[Griffiths Sec. 9.5, Chs. 10 & 12] 2023/06/08, 10:10am – 12:00am, 教師：張存續 Total score: 110

◇ Useful formulas: One A4 double-sided paper with handwriting.

1. (20%) Write the equations (if possible) and explain the following terms as clear as possible.

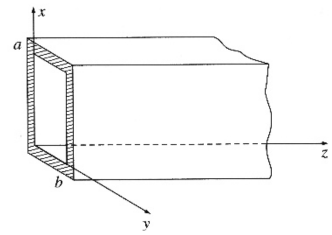
(a) The Lorentz gauge and the Coulomb gauge. (4%)

(b) Gauge transformations and gauge freedom. (4%)

(c) Hidden momentum (4%)

(d) The two postulates of the special relativity (4%)

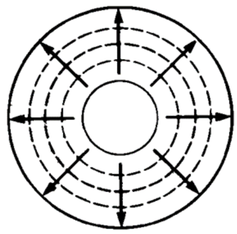
(e) Invariant quantity and conserved quantity. (4%)



2. (20%) A wave is propagating in a rectangular waveguide with TE_{10} mode. $B_z(x,z,t) = B_0 \cos(\pi x / a)\cos(kz - \omega t)$

(a) Find E_x and B_y? (10%) [Hint: Express in real components.] (10%)

(b) Estimate the cutoff frequency in GHz of the TE_{10} mode with width $a = 7.112$ mm and height $b = 3.556$ mm. [Hint: i.e., WR-28 waveguide] (10%)



3. (20%) A coaxial cable usually works in the TEM mode with the fields

$$\mathbf{E}(s,\phi,z,t) = \frac{A\cos(kz-\omega t)}{s}\hat{\mathbf{s}} \quad \text{and} \quad \mathbf{B}(s,\phi,z,t) = \frac{A\cos(kz-\omega t)}{cs}\hat{\boldsymbol{\phi}}$$

(a) Find the time averaged Poynting vector <**S**> and the energy density <*u*>. [Hint: Integrate over the cross section of the coaxial cable with the inner radius *a* and the outer radius *b* to get the energy per unit time and per unit length carried by the wave.] (10%)

(b) Confirm that the energy in the coaxial cable travels at the group velocity. (10%)

4. (25%) A point charge q moves in a circle of radius a at constant angular velocity ω. Assume the circle lies in xy plane, centered at the origin, and at time $t=0$ the charge is at (0, a), on the positive y axis.

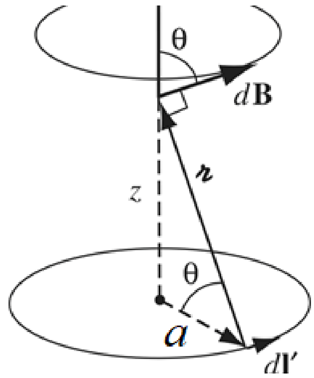
(a) Find the Lienard-Wiechert potentials for the points on the z axis. (8%)

(b) Calculate B_z on the z axis as functions of z and t. [Hint: $\mathbf{B}=\nabla\times\mathbf{A}$] (8%)

(c) Consider a circular loop of radius a, which carries a steady current I. The magnetic field at distance z above the center is

$$B(z)=\frac{\mu_0 I}{2}\frac{a^2}{(a^2+z^2)^{3/2}}$$

Compare the results shown in (c) with those in (b). (9%)



5. (25%)

(a) Show that ($\mathbf{E}\cdot\mathbf{B}$) is relativistically invariant. (8%)

(b) Show that ($E^2-c^2B^2$) is relativistically invariant. (8%)

(c) The relativistic transformation of EM fields between a rest frame K and a moving frame K' with velocity $\mathbf{v}$ and Lorentz factor γ are:

$$\begin{cases}\mathbf{E}'_{\|}=\mathbf{E}_{\|}\\ \mathbf{E}'_{\perp}=\gamma\left(\mathbf{E}_{\perp}+\frac{\mathbf{v}}{c}\times\mathbf{B}_{\perp}\right)\end{cases}\quad\text{and}\quad\begin{cases}\mathbf{B}'_{\|}=\mathbf{B}_{\|}\\ \mathbf{B}'_{\perp}=\gamma\left(\mathbf{B}_{\perp}-\frac{\mathbf{v}}{c}\times\mathbf{E}_{\perp}\right)\end{cases}$$

Find the velocity $\mathbf{v}_0$ such that the transverse electric components $\mathbf{E}'_{\perp}$ disappear. (9%)